Solutions to H.W. #1

1. $(-21, 23) - (x, 6) = (-25, y)$

$(-21-x, 23-6) = (-25, y)$

$-21-x = -25 \qquad 23-6 = y$

$x = 4 \qquad y = 17$

2. $(2,3,5) - 4i + 3j = (2,3,5) + (-4,3,0) = (-2,6,5)$

3. $(8a, -2b, 13c) - (52, 12, 11) = \frac{1}{2}(x, y, 2)$

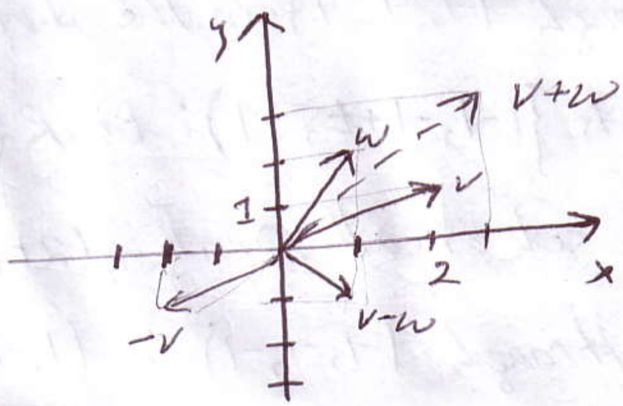
$(8a-52, -2b-12, 13c-11) = \frac{1}{2}(x, y, 2)$

$2(8a-52, -2b-12, 13c-11) = (x, y, 2)$

$(2[8a-52], 2[-2b-12], 2[13c-11]) = (x, y, 2)$

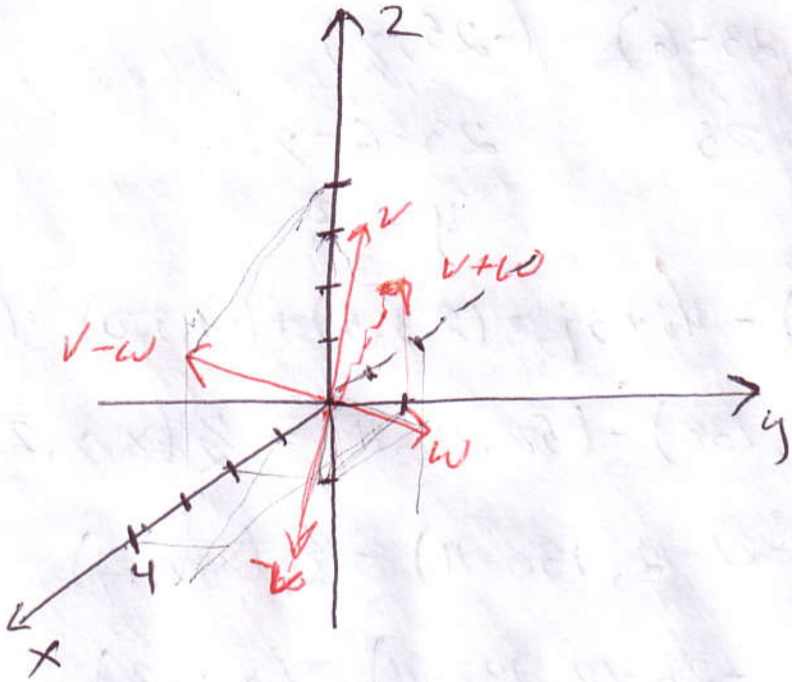
4. $v = (2,1)$, $w = (1,2)$, $-v = (-2,-1)$, $v+w = (3,3)$

$v-w = (2,1) - (1,2) = (1,-1)$



5. $v = (2,1,3)$, $w = (-2,0,-1)$, $-v = (-2,-1,-3)$

$v + w = (0,1,2)$ $v - w = (4,1,4)$



6. The plane spanned by $v = (2,7,0)$ & $w = (0,2,7)$ is ~~given by~~ the set of all points $P = \{(2t, 7t+2s, 7s) : s, t \in \mathbb{R}\}$

7. The line through $(-1,-1,-1)$ in the direction $\vec{j}$ is the set of all points $\{(-1, -1+t, -1) : t \in \mathbb{R}\}$.

8. This line is the set of all points $\{(2t, 2, 1-t) : t \in \mathbb{R}\}$

9. Line passing through $(-1,-1,-1)$ & $(1,-1,2)$ has direction $\vec{v} = (1,-1,2) - (-1,-1,-1) = (2,0,3)$

Hence it is the set of all points $\{(-1+2t, -1, -1+3t) : t \in \mathbb{R}\}$

10. $\{(-5+11t, -3t, 4-2t) : t \in \mathbb{R}\}$

11. The parallelogram with adjacent sides $\vec{v}$ and $\vec{w}$ is the set $\{\vec{v}t + \vec{w}s : s,t \in \mathbb{R}\}$

Hence, if $\vec{v} = \vec{i} + 3\vec{k}$ and $\vec{w} = -2\vec{j}$ the set is $\{(1,0,3)t + (0,-2,0)s : s,t \in \mathbb{R}\} = \{(t, -2s, 3t) : s,t \in \mathbb{R}\}$

12. The intersection of any line $L(t) = (x(t), y(t), z(t))$ with the coordinate planes xy, xz, and yz happens precisely when $z(t) = 0$, $y(t) = 0$, and $x(t) = 0$ respectively.

In the particular case when $L(t) = (3+2t, 7+8t, -2+t)$, $z(t) = -2+t = 0$ when $t = 2$. Hence the point of intersection with the xy plane is $\underline{L(2) = (7, 23, 0)}$

$y(t) = 7+8t = 0$ when $t = -\frac{7}{8}$. $\underline{L(\frac{-7}{8}) = (\frac{5}{4}, 0, \frac{-23}{8})}$

$x(t) = 0$ when $t = -\frac{3}{2}$ $\underline{L(\frac{-3}{2}) = (0, -17, -\frac{7}{2})}$

13. The line $v(t) = (2+t, -2+t, -1+t)$ intersects the plane $2x - 3y + z - 2 = 0$ iff $2(2+t) - 3(-2+t) + (-1+t) - 2 = 0$ for some $t \in \mathbb{R}$. $4 + 2t + 6 - 3t - 1 + t - 2 = 0$ or

$4+6-1-2+3t-3t = 0 \Rightarrow 4+3 = 0$ which is a contradiction.

Hence the line $v(t)$ does not intersect the given plane.

14. let $L(t) = (3t+2, t-1, 6t+1)$ and $v(s) = (3s-1, s-2, s)$.

Then these two lines intersect iff the equations

$$3t+2 = 3s-1$$
$$t-1 = s-2$$
$$6t+1 = s$$

have a solution. The middle equation implies that

$t+1=s$

substituting into the third equation, we get

$6t+1=t+1$ or $6t=t \Rightarrow t=0 \Rightarrow s=0+1=1$

substituting the solution $t=0, s=1$ into the 3 equations we get

$3\cdot 0+2 = 3\cdot 1-1$ ✓

$0-1 = 1-2$ ✓

$6\cdot 0+1 = 1$ ✓

Hence the solution $t=0, s=1$ is consistent in all three equations. It follows that the lines $L(t)$ and $V(t)$ intersect at the point $\underline{L(0)=(2,-1,1)}$.

15. Let $L(t)=(t+4, 4t+5, t-2)$ and $V(s)=(2s+3, s+1, 2s-3)$

then, just like in the previous problem, we must check whether the system of equations

$t+4=2s+3$

$4t+5=s+1$

$t-2=2s-3$

is consistent. The third equation implies that $t=2s-1$

substituting in the second equation, we get

$4(2s-1)+5=s+1 \Rightarrow 8s+1=s+1 \Rightarrow s=0 \Rightarrow t=-1$

checking:

$-1+4=2\cdot 0+3$ ✓

$4(-1)+5=0+1$ ✓

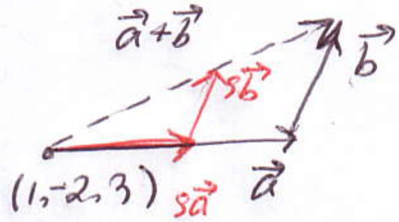
$-1-2=2\cdot 0-3$ ✓

Hence the lines intersect. The point of intersection is $V(0)=(3,1,-3)$.

16. The desired parallelepiped is the set of all points

$\mathcal{P} = \{(1,-2,3) + r\vec{a} + s\vec{b} + t\vec{c} \,;\, r,s,t \in [0,1]\}$

17.



the set of points lying on ~~edge~~ edge $\vec{a}$ can be described by the set $L = \{(1,-2,3) + s\vec{a} \,;\, s \in [0,1]\}$

For any fixed s, we extend a vector parallel to $\vec{b}$ with tail on $(1,-2,3) + s\vec{a}$. this parallel vector $t\vec{b}$ can be at most $s\vec{b}$ because it cannot reach beyond the edge $\vec{a}+\vec{b}$.

Hence the desired set is $T = \{(1,-2,3) + s\vec{a} + t\vec{b} \,;\, s \in [0,1],\, t \in [0,s]\}$